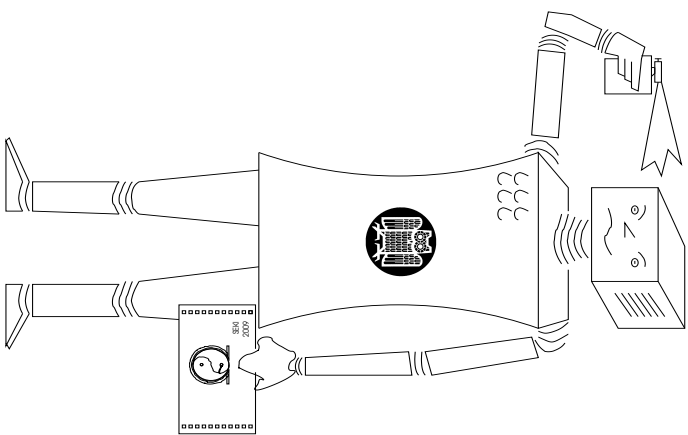


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Designing a GUI for Proofs –

Evaluation of an HCI Experiment

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Abstract

Often user interfaces of theorem proving systems focus on assisting particularly trained and skilled users, i.e., proof experts. As a result, the systems are difficult to use for non-expert users. This paper describes a paper and pencil HCI experiment, in which (non-expert) students were asked to make suggestions for a GUI for an interactive system for mathematical proofs. They had to explain the usage of the GUI by applying it to construct a proof sketch for a given theorem. The evaluation of the experiment provides insights for the interaction design for non-expert users and the needs and wants of this user group.

1 Introduction

Human-computer interaction (HCI) is the interdisciplinary study of interaction between people (users) and computers. Its main goal is making computers more user-friendly and easier to use. HCI is concerned with methodologies and processes for designing interfaces, with methods for implementing interfaces, with techniques for evaluating and comparing interfaces, with developing new interfaces and interaction techniques and with developing descriptive and predictive models and theories of interaction [9].

More often than not, user interfaces for theorem provers are developed as a mere add-on to the main proving engine. The result is an interaction design suitable for proof experts only. As example of such an interaction design consider the graphical user interface (GUI) LOUI [11] of the Ω MEGA system [10] developed in our group in Saarbrücken, Germany: proof presentation as well as the functionalities provided to the user are tailored to support an expert of the underlying proving system.

Such an expert-oriented interaction design seems to be sufficient as long as the only users of interactive theorem provers are trained and skilled users, which are familiar with logic notations and the particularities of underlying proof engines. However, there are other application domains of theorem provers for which the user group consists of non-experts. As example domain consider the usage of mathematical systems as cognitive tools in a learning environment. Currently, the

MIPPA project [8] develops an interactive learning system for mathematical proofs, which will be connected with the ACTIVEMATH learning environment [5]. The underlying proof engine is the MULTI proof planner [7], which is part of the Ω MEGA system (see [8] for the motivation of using MULTI as proof engine for this task).

In order to make theorem proving systems useful for a larger clientele we have to ask “What are the needs and wants of non-expert users?”. In particular, the development of a suitable interaction design is a crucial point since: *Developers often see the functionality of a system as separate from the user interface, with the user interface as an add-on. Users, however, do not typically make distinctions between the underlying functionality and the way it is presented in the user interface. To users, the user interface is the system. Therefore, if the user interface is usable, they will see the entire system as usable* (quoted from [3]).

In order to develop a system that satisfies the needs and wants of envisioned users and supports the users in achieving their tasks, interaction design principles [9] suggest an iterative development process with several interleaved design, experiments, and evaluation phases of the user interface. To start with the interaction design for a GUI of an interactive theorem prover for non-expert users, whose main interest is to prove a mathematical theorem in an adequate way (whatever the underlying proof engine or calculus is), we conducted a paper and pencil experiment with students. The students had to develop a GUI for an interactive theorem prover with provided material (paper, pencils of different colors, etc.) and to use this GUI to construct a proof sketch for the theorem “ $\sqrt{2}$ is irrational”. They were free to invent and use all functionalities of the assumed underlying proof system they liked. You can find the original exercise and fotos of the proposals in appendix A and B.

This paper describes the experiment and its results. In particular, the evaluation of the student proposals provides basic insights into wants and needs of non-expert users when interacting with a system for constructing mathematical proofs.

In order to avoid some misunderstandings *ab initio*:

- The experiment is not about the evaluation of a given design with users. Rather, the users were asked to freely invent their own proposals.
- The students represent only one target group of the MIPPA system: students of computer science, mathematics, or engineering. With other target groups (e.g., pupils) similar experiments have to be conducted.
- In an educational context as in the MIPPA project the main question behind the interaction design (and anything else) actually is “How can learning be supported?” The context of the experiment and this paper, however, is restricted to the more general question of wants and needs of non-expert users.

2 The Experiment

As non-expert users we invited 8 students participating a seminar about proof planning [2, 6] at the computer science department of the Saarland University in Saarbrücken, Germany. From their studies all students had a profound background in computer science and mathematics as well as basic knowledge about logic. Because of the seminar the students were also familiar with notions such as *tactic* and *method*. Moreover, during the seminar they were briefly introduced to the Ω MEGA system and its GUI LOUI. In order to stimulate interchange of ideas the 8 students were grouped into 4 pairs, which are referred to as group A, group B, group C, and group D in the remainder of the paper.

In the following, we first detail the phases of the experiment and then describe the instructions provided to the students.

2.1 Sequence of Actions

The experiment consisted of five phases:

1. The students were provided with the instruction material (see section 2.2), which they had to read carefully. Afterwards, they could ask questions. For this phase 15 minutes were scheduled.
2. Each group had to prepare their GUI proposal and a presentation of how to use their GUI to prove the example theorem “ $\sqrt{2}$ is irrational”. To do so, the students were supplied with the following basic material: paper sheets of different sizes, pencils of different colors, scissors, glue, rulers. For this phase 120 minutes were scheduled.
3. Each group had to give a 15 minute presentation of their proposal. All presentations were recorded with a video camera.
4. After the presentations the students had another 10 minutes to reflect again their own proposal as well as the proposals of the others.
5. In a final discussion the students should point out what elements of the different proposals they liked or disliked.

2.2 Instructions

The following is a brief version of the instructions provided to the students.¹

¹Because of the background of the students it was possible to precisely describe their task in the experiment. For other target groups such as pupils without at least some ideas of the construction and arrangement of mathematical proofs this might turn out to be a much harder task.

The Example Theorem The example theorem “ $\sqrt{2}$ is irrational” was given to the students together with a brief proof sketch: Assume that $\sqrt{2}$ is rational. Then, there are two integers n, m that satisfy $\sqrt{2} = \frac{n}{m}$ and that have no common divisor. From $\sqrt{2} = \frac{n}{m}$ follows that $2 * m^2 = n^2$ (1), which results in the fact that n^2 is even. Then, n is even as well and there is an integer k such that $n = 2 * k$. The substitution of n in (1) by $2 * k$ results in $2 * m^2 = 4 * k^2$ which can be simplified to $m^2 = 2 * k^2$. Hence, m^2 and m are even as well. This is a contradiction to the fact that n, m are supposed to have no common divisor.

The Task The students were asked to develop a proposal for a GUI for a system to interactively prove mathematical theorems and to apply this GUI to prove the given example theorem. In particular, they were instructed to assume an underlying system that can provide all functionalities they would like to use to achieve their task. However, they were free to invent and use all functionalities of the assumed underlying proof system they liked. There were no instructions or limitations for the design.

For their presentation the students had to use the given material to prepare several states of the GUI, which cover several states of the proof of the example theorem. To construct the proof they should make use of two different ways for manipulating a proof under construction: *operator-based* proof development and *island-based* proof development. The former notion means that the system provides operators for proof manipulation, which can be used during the proof construction. Such an operator has some input and produces some output, which is introduced into the proof. For instance, consider an operator for definition unfolding. When applied to the occurrence of a defined concept such as “is rational” in a proof assumption, the application of the operator derives a new assumption in which the occurrence of “is rational” is replaced by its definition. Island-based proof development means that the user can freely introduce steps into the proof by inventing new statements, so called *islands* and their relations to other statements in the proof (in order to indicate from which other statements the island is supposed to follow or which other statements are supposed to follow from the island).

Presentation We were interested, in particular, in the GUI features and presentations the students would introduce as well as which functionalities they would demand from the underlying proof system. Moreover, we were interested in their motivations and how they would argue for and explain their proposals. Hence, the students were asked to point out in their presentations the following points:

- underlying ideas and the motivation of the group,
- the organization of their GUI,
- presentation of proofs and current proof status,
- application of operators,
- and introduction of island steps.

Data				Lexicon Database Help		
Text	Formulae	CaseSplit	Custom Step			
Theorem $\sqrt{2}$ is irrational. <u>Proof:</u>					Sugg.	Op.
Check Auto <input type="radio"/> <input type="radio"/> <input type="radio"/>						

Figure 1: GUI proposal (Group A).

3 Student Proposals and Discussion

This section first briefly describes the different proposals presented by the four groups and then gives a brief account of the discussion following the presentations. The descriptions of the student proposals are structured wrt. the five presentation points introduced in the previous section.

Not surprisingly wrt. the free setting of the experiment, many details of the single proposals stay unclear. Moreover, the experienced reader (i.e., the proof expert) may detect some inconsistencies within the proposals and the arguments of the students. However, both proposals and arguments provide interesting insights into the world of thought of the students.

3.1 Group A: Text-based and Operator-Centered Approach

General Idea The proof is constructed and presented in an accustomed manner “as taught at school”. Therefore, the group argues for a linear proof presentation mixing both text and mathematical formulae as main presentation format (see Proof Presentation).

GUI Window The GUI, see Figure 1, is divided into three parts: two bars with menus and buttons at the top, a presentation area of the current proof state and a choice area for proof techniques and operators.

The upper bar provides pull-down menus to access standard functionalities such as “load” and “save” as well as for lexicon, database, and help access. The lexicon menu allows to browse definitions, theorems or operators. The database menu allows to retrieve example proofs, which are stored in a database (e.g., proofs of “similar” problems). The help menu provides help functions

as known from standard software tools.

The lower bar consists of buttons for specific proof manipulations: a custom step button for island introduction, a case-split button, a text button for the introduction of text parts and a formulae button for the introduction of formulae.

The area for the choice of proof techniques or operators contains a tab for suggestions and a tab for operators (see Operator Application).

Proof Presentation The group favors linear and primarily text-based proof presentation. At the top are theorem and the assumption statements. New proof statements are added below independent from whether they result from forward reasoning from the assumptions or from backward reasoning from the conclusion. In the statements logical connectives and quantifiers are omitted and only mathematical formulae “as known known from school” are allowed. For the example “ $\sqrt{2}$ is irrational” a presentation looks like this:

Theorem: The square root of 2 is not rational.
Proof: We assume, that the square root of 2 is rational.
 Then there exist two numbers n and m , being coprime,
 such that $\sqrt{2} = \frac{n}{m}$ holds.
 Then $2 \times m^2 = n^2$ holds.
 ...

The underlying system is supposed to continuously check whether the current proof under construction is correct. Feedback of this check is provided via a traffic light in the proof presentation area. A green light shows the checked correctness of the current state, a yellow light the unchecked state. A bright red light indicates that the underlying system was not able to check the correctness without detecting explicit errors, whereas a dark red light signs the explicit detection of errors.

Another demanded function of the underlying system is the automated completion of the proof under construction. This functionality is accessed via an auto button in the proof presentation area. For automatically closed gaps the completed proof parts should be displayed in the same text-based manner as the other proof parts.

Operator Application The names of the operators are listed alphabetically in the operator tab. Clicking an operator name opens a small dialog window showing the pattern of the input arguments of the selected operator, see Figure 2. The user marks statements in the proof and drags them into the window to fill the input arguments. The underlying system immediately checks whether the arguments are a suitable input for the operator. Feedback of this check is given in form of traffic lights as well as by error descriptions. The successful application of an (instantiated) operator results in new text-based statements in the proof under construction.

To support the application of an operator the underlying system provides so-called *suggestions*. When the user clicks an operator name in the operator tab, then the system computes either full or partial instantiations of the selected operator. The user can access the computed suggestions via the suggestion tab, in which the suggestions are ordered wrt. the significance the system assigns to them.

Operator:		Indirect		?
Forward		Backward		
<p>Assumptions:</p> <div style="display: flex; justify-content: space-between; align-items: flex-start;"> <div style="width: 30%;"> <p>...</p> <p>...</p> </div> <div style="width: 30%; text-align: center;"> <div style="border: 1px solid black; padding: 2px 10px; margin-bottom: 10px;">Pick</div> <div style="border: 1px solid black; padding: 2px 10px;">Pick</div> </div> <div style="width: 10%; text-align: center;"> <div style="border: 1px solid black; border-radius: 50%; width: 15px; height: 15px; margin-bottom: 5px;"></div> <div style="border: 1px solid black; border-radius: 50%; width: 15px; height: 15px; margin-bottom: 5px;"></div> <div style="border: 1px solid black; border-radius: 50%; width: 15px; height: 15px; margin-bottom: 5px;"></div> <div style="border: 1px solid black; border-radius: 50%; width: 15px; height: 15px; margin-bottom: 5px;"></div> <div style="border: 1px solid black; border-radius: 50%; width: 15px; height: 15px; margin-bottom: 5px;"></div> </div> <div style="width: 30%; text-align: right;"> <div style="border: 1px solid black; padding: 2px 10px; margin-bottom: 10px;">Apply</div> <div style="border: 1px solid black; padding: 2px 10px;">Cancel</div> </div> </div>				

Figure 2: Operator application (Group A).

Island Introduction To introduce an island step the user hits the custom step button. This opens a dialogue window similar to the window for the operator application. The difference is that no operator name and no pattern is displayed. Rather, the user inputs the island statement via an input editor and determines the number of input arguments by dragging and dropping as many statements as wanted. Thereby, input statements are either assumptions or goals, turning the island into a new assumption or goal respectively.

On demand the underlying system checks new islands. That is, the system tries to prove either that an assumption island follows from the specified assumptions or that the specified goals follow from a goal island. The system issues feedback on this check to the user.

Miscellaneous Proof completion might be detected by the underlying proving system automatically or be indicated explicitly by the user.

3.2 Group B: Bridge Building Metaphor

General Idea The group uses a bridge as metaphor for theorem proving: construct a bridge between assumptions and conclusion. Moreover, the group invents supporting visualizations such as icons for operators and drag & drop techniques for the bridge construction (see Operator Application).²

GUI Window The GUI consists of two parts, see Figure 3. The main area on the left is used for rendering proofs as a bridge. The area on the right comprises a control and an operator button area as well as an information and a help area.

²The bridge building metaphor has already been described by Polya[4] from a pedagogical point of view.

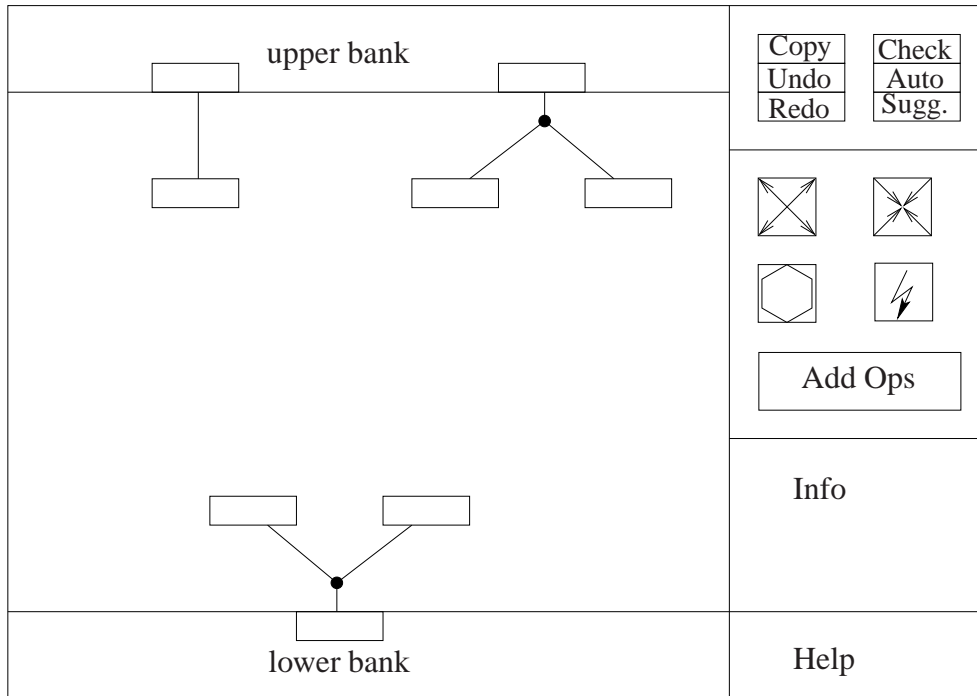


Figure 3: Bridge Building GUI (Group B).

The control button area provides “usual” control functionalities such as undo, redo, automatic proof completion and suggestions. Moreover, by hitting a “Finished” button, the user can indicate, that he expects the proof to be finished. In the operator button area most frequently used and relevant (for the current problem) operators are depicted by icons. The information area shows feedback that the system provides to the user. The user can request help from the system by dragging objects (e.g., operator icons, buttons, statements etc.) onto the help field. Then, a feedback window opens and provides specific information on the object.

In contrast to other proposals, the group invents icons instead of names for operator identification. Each operator is associated with a “meaningful” icon, for instance, a lightning bolt may be used for contradiction. Other examples are given in Figure 4. “Expansion” unfolds the definition of a selected statement whereas “Collapse” has the opposite effect by contracting an unfolded definition. By hitting the Island button new islands can be introduced. Experienced users may introduce further icons for operators.

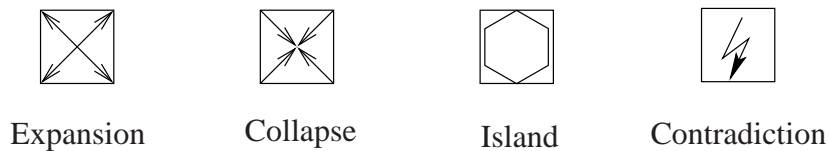


Figure 4: Operator iconization by Group B.

Proof Presentation The proof state is presented as bridge under construction between the assumptions and the conclusion. Bridge nodes consist of nested statements. Edges between the nodes denote operator applications or island introductions. In the former case, the edge is labeled with the operator icon.

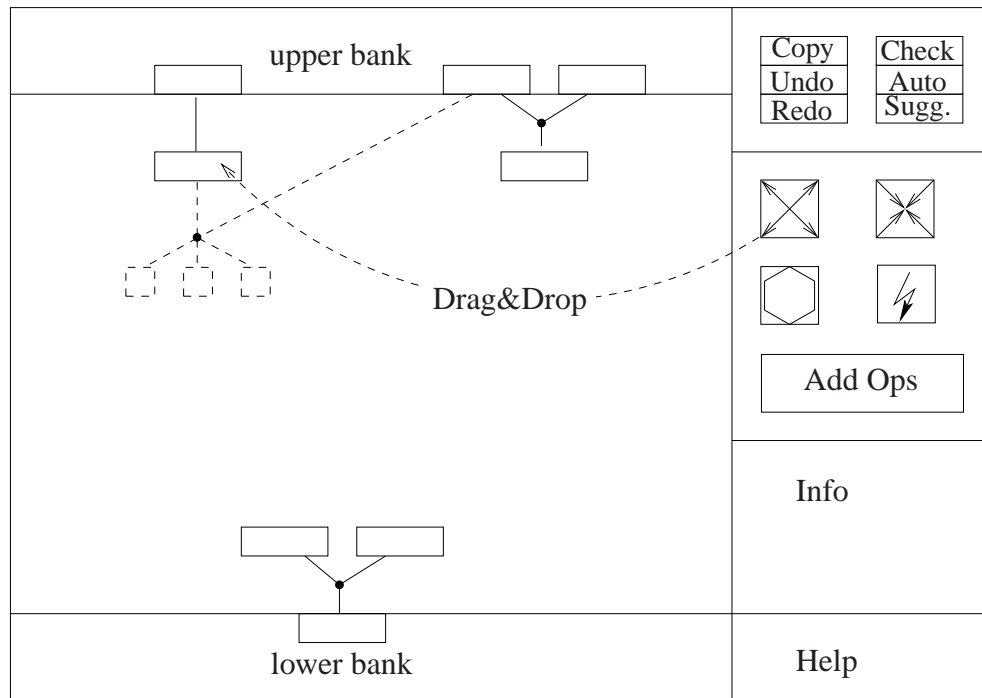


Figure 5: Operator application (Group B).

The students pointed out that, if proofs become too large, then some minimization or shrinking mechanism might be useful, which allows to replace whole proof parts by a single node or label.

Operator Application To apply an operator the user drags the operator icon and drops it onto a proof statement in the proof state presentation (see Figure 5). This proof statement is one of the inputs for the operator. Next, new nodes are inserted into the bridge, which represent the output of the operator. The new nodes become connected by an edge with the initial statement. If more input is necessary to apply the operator anchors appear at the new link that the user has to drag & drop onto further input statements.

If the initial drag & drop of the operator icon onto a statement or the drag & drop of an anchor onto other statements causes an error (i.e., if the selected statements are not suitable as input for the operator), then the system provides error feedback in the information field.

Island Introduction An island is inserted by dragging the island icon and dropping it “somewhere into the river”. Using an input editor the user specifies the island statement, which appears “in the river” as a new bridge node. This new node is initially not connected to any other nodes. The user connects it by dragging and dropping anchors that appear at the new island node.

Note that the island approach of group B conceptually differs from the island approach introduced by group A. Group A allows only islands that are directly connected with statements already in the proof. Actually, this results in headlands instead of real islands. As opposed thereto, the approach of group B allows for real islands, since the island statements do not have to be connected immediately with other statements.

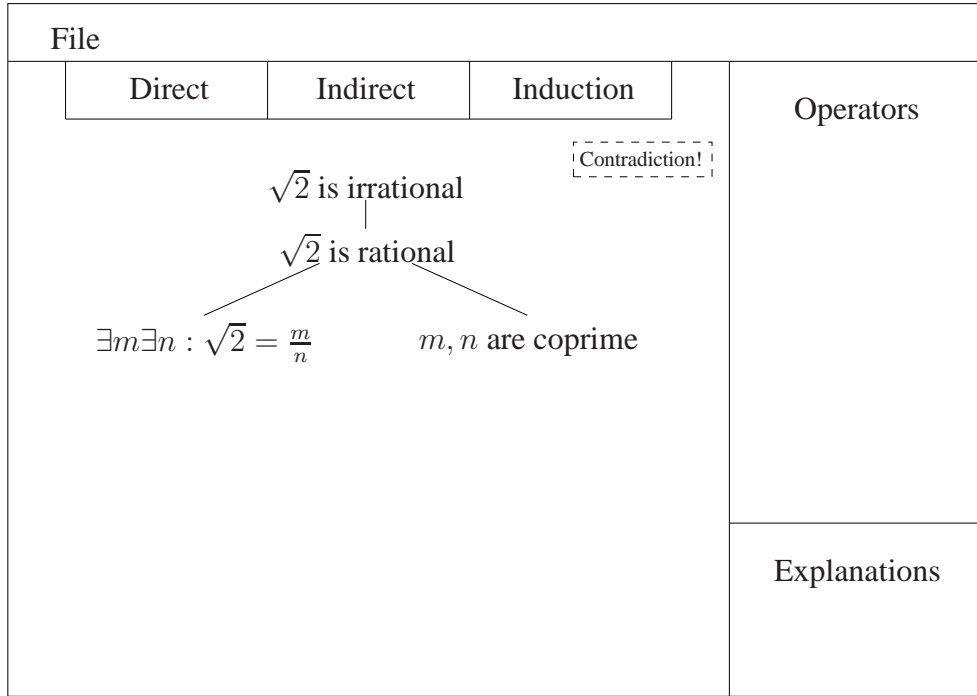


Figure 6: Masking operator names (Group C).

Miscellaneous The group suggested more features such as copying of proof parts and automatization of primitive steps. Moreover, a dialog in the beginning should ask the user about the general proof technique to employ, for instance, a direct, an indirect or a proof by induction.

3.3 Group C: Masking Operator Names

General Idea This group favors a top-down style for proof presentation “as taught at school or university”. The proposal differs from other proposals by its omission of operator names. Operators are expressed by their input and output (see Operator Application), since “the main goal is not to learn some operator names or icons but to understand what follows from what”.

GUI Window The GUI (see Figure 6) has a bar with pull-down menus at the top providing functionalities known from standard software such as loading and saving of proofs etc. The rest of the window is divided into two parts. The left part presents the proof state whereas the right part consists of a field for operator suggestions and a field for explanations of operator applications (the Operator Application).

On the top of the proof presentation part is a bar with buttons for “standard proof techniques” such as direct or indirect proof. This bar is built-up dynamically. For instance, when the user decides for an indirect proof, a contradiction button is added to the bar. The meaning of this new button is that the user can finish the proof by specifying two contradictory statements in the proof under construction.

Proof Presentation. Proofs are presented as trees of statements. However, the proof tree is constructed in a top-down manner similar to the approach of group A. New statements are introduced

as nodes below the statements from which they are derived and connected by edges. However, such edges do not denote that certain statements are supposed to be logical consequences of other statements. Rather, they are “storytellers” of the kind: “next do . . . to get . . .”. Assumptions and goals are not explicitly separated in the tree.

Operator Application For operator application group C uses also a suggestion mechanism as introduced already by group A. When the user clicks a statement in the proof, the underlying system computes suggestions of operator applications with this statement as input. These suggestions are displayed in the operators field on the right. Neither names nor icon representations of the operators of the suggestions are displayed. Rather, the output of the suggestion is displayed. If the user clicks a suggestion (via its output), then detailed information on the underlying step is given in the explanation field, in particular, an explanation (justification) for the derivation of the output from the input.

Note that the suggestion concepts of group A and group C differ wrt. their initialization. The *operator-oriented* suggestion mechanism of group A is initiated by selecting an operator for which suggestions are computed. As opposed thereto, the *statement-oriented* suggestion mechanism of group C is initiated by selecting a statement for which suggestions with different operators can be computed.

Island Introduction Similar to the proposal of group A also this proposal reduces islands to headlands. Headlands are introduced by marking a statement and clicking the add button in the suggestion field, which opens an input editor to enter a new statement. The new statement is inserted into the proof and is linked with the initially chosen statement. Further links can be added.

Miscellaneous The group points out that some simplifications like quantifier elimination should be done automatically by the system.

The explanation field can be used in different ways. When the user clicks a suggestion (see above), then an explanation of the step offered by the suggestion is given. If the user clicks a statement in the proof, then an explanation of the derivation of the statement from its parent nodes is given.

3.4 Group D: Structuring with Notebooks

General Idea The main idea of this approach is to construct and present any proof in the accustomed pencil and paper manner “as taught at school”. This implies a linear line of reasoning as well as the ability of extracting subproofs or conducting several proof attempts. This is realized by a notebook concept, where each tab corresponds to a subproof. Copying proof parts between tabs should be supported.

GUI Window The GUI (see Figure 7) has two bars at the top. The upper bar provides standard pull-down menus “as known from standard software tools”. The second bar consists of icon buttons to access standard functionalities such as loading and saving of files, copy and paste of

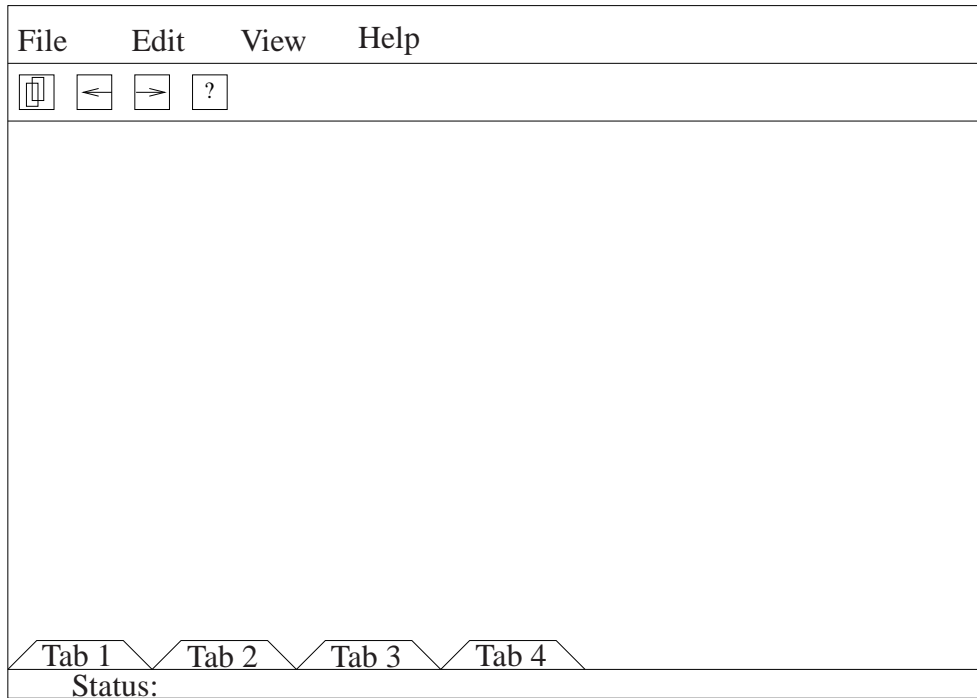


Figure 7: Structuring with Notebooks (Group D).

objects, view and help directly “as known from standard software tools”. At the bottom of the GUI is a status line for giving feedback. The main field of the GUI consists of different tabs containing different proof parts or proof attempts.

Proof presentation. Proofs are presented in a linear style, where nested statements are connected by arrows (\Leftrightarrow , \Leftarrow , \Rightarrow). The arrows are labeled by operators and denote different kinds of mathematical consequence relations, e.g., equivalence or implication. Figure 8 gives an example presentation of a proof. The students emphasized that statements should be freely arrangeable and relocatable.

Operator Application When the user clicks a statement in the proof, an operator dialog window is opened (see Figure 9). The window offers lists of most recently used and most popular operators (e.g., performing induction, indirect proof, or introducing case splits) in which operators are displayed by their names. When the user selects an operator, then this operator is applied with the initially selected statement as input. If additional input is necessary, then the user has to provide it by drag & drop of further statements into the dialog window.

The user can browse the complete list of available operators. Since this list might be long, supporting search functionality is provided. This allows for full text search in operator descriptions and for synonyms rather than pure name-based operator identification. For instance, the search with the inputs *definition expansion*, *expansion* or *explanation* results in the operator for definition expansion.

Island Introduction Similar to group B this approach supports real islands. By clicking somewhere in the proof development area an island node can be freely located. The island statement

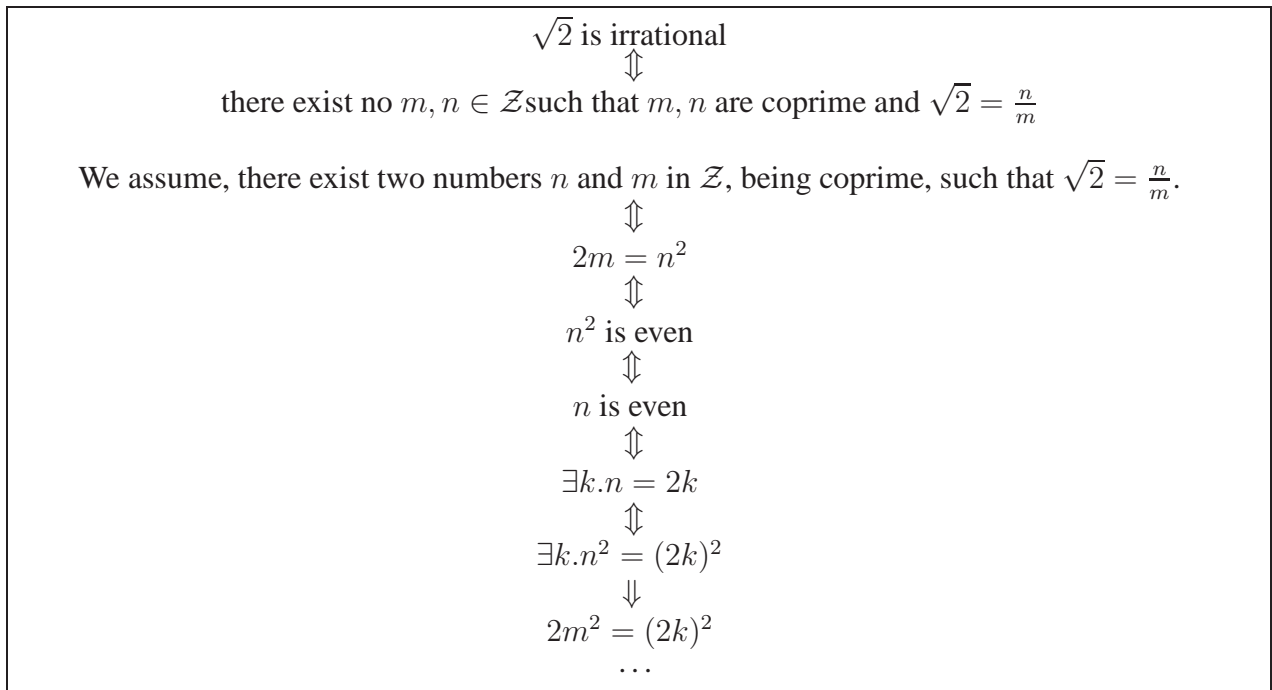


Figure 8: Proof presentation (Group D).

Operators	
<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px; text-align: center;"><i>Search input</i></div> <div style="border: 1px solid black; height: 150px; margin-bottom: 10px;"></div> <div style="margin-top: 20px;"> Most popular: <ul style="list-style-type: none"> - Induction - Indirect - Case split </div>	Recently used: <ul style="list-style-type: none"> - Substitution - Transitivity

Figure 9: Operator selection (Group D).

is specified via an input editor. When introduced, islands are initially not connected with other statements. The user can freely introduce connections. The group called such a connection link with an island a “declaration of intent”.

3.5 Student Discussion

After reflecting all proposals we asked the students to discuss the benefits and drawbacks of the different proposals. The following is a summary of the most interesting points.

- Members of group A pointed out again the general appropriateness of linear and textual proofs. However, they realized the drawbacks of their proposal wrt. island introduction and distinction of assumptions and goals.

Similarly, also the members of group D admitted the drawbacks of their linear proof presentation, in particular, the lack of separation of assumptions from goals, which both occur at any place in their proofs, as well as the distinction of forward and backward reasoning.

After reflection, all students considered the bridge metaphor invented by group B as more appropriate for organizing and overviewing the proof state and progress.

- All students agreed that the combination of context-sensitive suggestion mechanisms (see group A and C), operator filters (most recently, most popular) and full-text synonym search (see group D) would form a powerful operator selection widget. Moreover, all students considered freely arrangeable and relocatable statements as well as the notebook concept suggested by group D as very helpful.
- The supervisors mentioned that obviously each group avoided logical notation. The students argued that (full) logical notation is not what they are familiar with in mathematical theorem proving from school and university. Moreover, they had objections against dealing explicitly with logical connectives and quantifiers in their proofs.

4 Evaluation: User Wants and Needs

As evaluation of the experiment we analyzed the proposals and the discussion of the students for interesting presentations and features in the GUI as well as for demanded functionalities of the assumed underlying proof systems. In particular, we were interested in points more or less accepted by all students.

4.1 Presentations and Features in the GUI

The students naturally used many features offered by standard software tools, for instance, pull-down menus, copy & paste of content/proof parts and drag & drop of GUI objects. Such features reflect the experience of the students with standard software as well as the state-of-the-art in GUI design. More specific, for proof construction the students suggested and used the following presentations and features.

Nested Statements Each group used – at least implicitly – nested statements composed of text pieces or formulae. Sub-statements should be accessible for drag & drop and operator application.

Text-Based Statements There was a common agreement about omitting pure logical notation with quantifiers and connectives. All proposals used some combination of verbalization and mathematical formulae, such as “There exists an x , such that ...”.

Input Editor All groups demanded an input editor for mathematical formulae and composed statements.

Proof State Presentation Apart from group A, all groups presented proof states as some form of graph or tree where nodes are labeled with statements and edges are labeled with operators, island connections or mathematical consequence relations such as \Leftarrow , \Rightarrow , \Leftrightarrow .

In the discussion, the students picked up the bridge construction metaphor very quickly and accepted its benefits: clear separation between assumptions and goals as well as clear separation between forward reasoning and backward reasoning.

Notebook In the discussion, the students agreed that the notebook concept introduced by group D is useful to structure a complex proof into subproblems or to allow for different proof approaches. They pointed out, however, that such a concept has to be accompanied by a copy & paste mechanism for proof parts.

Operator Identification and Selection In their presentations all groups introduced special GUI presentations for operators they considered to be special such as indirect proof, case-split introduction or induction.

Moreover, the students agreed that an identification of the operator by name only is not appropriate. They considered iconization of operators as useful but limited approach (assigning every operator a suitable symbol might be problematic).

Argument Selection There was a common agreement that drag & drop is a necessary support for the selection of the arguments of an operator. That is, when arguments have to be selected for an operator application, then these arguments should be dragged in the proof presentation and dropped into some operator application dialog window.

4.2 Demanded Functionalities of Underlying System

All proposals comprised and demanded many functionalities known from standard software tools such as loading and saving of documents/proofs, undo function for proof steps and help function (e.g., explanations of the operators). However, the students also demanded interesting functionalities specific for the construction of mathematical proofs.

Support for Operator Application Two of the groups invented some form of context-sensitive suggestions. Depending on whether the user marks an operator or a statement for application (operator-oriented vs. statement-oriented) the system computes and provides (suggests) completely or partially instantiated operators. This should free the user from the often laborious task to select all arguments of an operation application.

Check Operator Arguments When the user provides the input arguments for an operator, then the system should check whether the provided input is consistent with the input specification of the operator. If this check fails, then the system should provide detailed feedback on the reason of the failure.

Check Proof The system should check the correctness of the proof under construction (either continuously or on demand). It should provide feedback to the user about the result of this check: proof correct, check failed, errors detected. In particular, the detection of errors should result in feedback on both the detected errors and their cause.

Automation Support All groups demanded some automated proof construction support, for instance:

- The system should perform “simple” steps automatically, such that the user does not have to bother with them. No group detailed the notion of “simple”.
- The system should be able to verify introduced islands. This corresponds to the computation of a subproof for the island.
- The system should be able to complete on demand gaps in the proof under construction automatically. If the system fails, it should provide feedback on the reason of the failure.

Hints The system should provide hints on how to proceed, for instance:

- Depending on the current “proof strategy” the system should give general advice. For instance, when the user constructs an indirect proof, then the system could provide “Derive a contradiction!” as a general advice.
- When providing suggestions, the system should rank these suggestions such that “the best ones” are ranked first.
- When a user interaction results in a failure, then the system should offer specific hints and guidance how to overcome the failure. For instance, when the user specifies input arguments not suitable for an operator, then the system could suggest suitable input arguments.

Feedback All students pointed out that the provision of feedback as response to user interactions is very important. Feedback should follow both successful user interactions – confirming the user interaction – and failing user interactions – explaining the problem as detailed as possible.

Retrieve Examples from Database The system should support the retrieval of data from a proof database. In particular, it should support the retrieval of proofs for “similar” problems as well as the retrieval of example applications of selected operators. This allows the study of successful proofs and successful operator applications. None of the groups elaborated the notion of “similar” problems.

5 Conclusion

In this paper, we described a paper and pencil HCI experiment conducted with graduate computer science students. The aim of the experiment was to gain basic insights into the wants and needs of non-expert users of a system for mathematical proof, i.e., users that are not particularly trained wrt. a certain theorem proving system and its functionalities.

In the experiment the students were asked to invent a GUI for an interactive system for mathematical proof. They had to explain their GUI structure and the used functionalities of the assumed underlying system with the example problem “ $\sqrt{2}$ is irrational”. In a subsequent discussion the students were asked to reflect their own proposal as well as the other proposals and to point out features and functionalities they particularly liked or disliked.

Among the (more or less) commonly agreed GUI features and useful functionalities are:

- statement presentation that avoids logical notation and mixes textual presentation with mathematical formulae,
- a graphical presentation of proof states that supports distinguishing goals and assumptions as well as forward and backward reasoning,
- rich system support for performing proof steps (e.g., computation of suggestions),
- rich system feedback on user interactions (e.g., explanation of failures),
- automated proof construction support (e.g., simple steps should be done automatically),
- provision of hints (e.g., hints to overcome a failure).

Moreover, the students naturally demanded functionalities known from standard software, e.g., loading and saving of proofs and availability of an undo function. They also naturally used state-of-the-art GUI techniques such as copy & paste and drag & drop.

An interesting observation was that, initially, a majority of the students argued for a linear character of proofs, not only of the final proof but also of the intermediate proof states during the construction. They repeatedly stated that such a linear form of proof and proof state presentation is their experience from school and university lectures and tailored their GUI presentations to this form. This observation is consistent with empirical studies that suggest that student’s deficiencies in their mathematical competence with respect to understanding and generating proofs are connected with a basic misunderstanding of the theorem proving process. Typical presentations of proofs in math books and lectures present proofs as linear constructs that derive the conclusion from the assumptions. This results in the shortcoming of learners understanding of theorem proving as a highly non-linear and hierarchical search and construction process. Therefore it is not surprising, that the students picked up the bridge building metaphor as a more adequate way of proof construction, since it enforces and clarifies proof states and proof reasoning.

Based on the observed user wants and needs we will develop a GUI prototype. In order of achieving usability, especially for our non-experts target group, we intend to evaluate our prototype theoretically according to the cognitive dimensions framework [1] as well as experimentally with target users. To focus on our concrete task in the MIPPA project (see section 1) we will also conduct further experiments explicitly focusing on an interactive system for learning mathematical proof.

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A Original German Instructions for the Students

Seminar Experiment

16.07.2004

Hallo Liebe Studenten und Willkommen zu unserem Experiment!

1. Einleitung

Um was geht es hier und heute?

Ihr habt fleißig an unserem Proof Planning Seminar teilgenommen und auch ein wenig mit Ω MEGA und seinem GUI LOUI gearbeitet. Nun wollen wir etwas sehr Schwieriges von euch: Ihr sollt für die nächsten Stunden alles vergessen, was ihr über Proof Planning, Ω MEGA und LOUI wisst. Denn wir wollen mit euch ein Experiment durchführen an Hand dessen Resultate wir ein Graphical User Interface entwickeln wollen mit dem Studenten mathematische Beweise führen sollen. Ein dahinter liegendes Beweis-Tool soll Support und Funktionalitäten zur Verfügung stellen. Aber das GUI soll auf keinen Fall speziell für Ω MEGA entwickelt werden. Daher sollt ihr eben nicht von euren Ω MEGA Erfahrungen ausgehen, sondern sollt eurer Kreativität freien Lauf lassen und ein GUI sowie eine Beweisdarstellung nach eurem Geschmack entwerfen. (Müssen mathematische Aussagen in Formeln kodiert sein? Muss ein Beweis als Baum dargestellt werden? Was hat euch an LOUI und Ω MEGA besonders gestört? Wie würdet ihr gerne Beweise aufbauen und manipulieren?).

Wie soll das Ganze ungefähr ablaufen?

Wir geben euch eine Beweisaufgabe vor, die in einer bestimmten Art und Weise bearbeitet werden soll. Desweiteren geben wir euch Material vor: Stifte, Papier, Kleber, Lineale sowie vorgefertigte GUI Elemente wie verschiedene Arten von Windows und Menues. Ihr werdet in Zweier-Gruppen eingeteilt und sollt dann jeweils ein GUI entwerfen um damit die beschriebene Aufgabe zu bearbeiten. GUI entwerfen heißt: Nehmt vorgefertigte GUI Elemente oder entwickelt und fertigt euch selbst GUI Elemente, arrangiert diese auf den ausgegebenen DIN A2 Blättern und spielt dann durch, wie die Aufgabe in eurem GUI abläuft. Für diese Konzeptions- und Vorbereitungsphase habt Ihr ca. 70 Minuten Zeit. Anschließend stellt jede Gruppe ihren Ansatz am Flipchart vor (jeweils bis zu 15 Minuten) und wir diskutieren gemeinsam die Ansätze.

2. Material

Als Basismaterial stehen euch zur Verfügung:

- DIN A2 Blätter zum Arrangieren des GUI's
- Stifte in verschiedenen Farben
- Kleber

- Schere
- Lineal
- DIN A3 und DIN A4 Blätter (um euch beliebige GUI Elemente selbst daraus zu schneiden)

Folgende vorgefertigten GUI Elemente liefern wir euch schon mal mit:

- verschiedene Windows:
 - Standard Leer-Window (ergänzt, was immer ihr wollt)
 - Listen Windows
 - Text Windows
 - Tabellen Windows
 - Dialog Windows
- Kontext Menues

Hinweis: Das gesamte vorgegebene Material ist eigentlich nur dazu da euch Impulse zu liefern. Wenn euch das vorgegebene Material nicht gefällt bzw. es nicht ausreicht um eure Vorstellungen zu realisieren, dann dürft ihr euch selbst aus dem Basismaterial basteln, was immer ihr wollt!

3. Aufgabe

Die Beweisaufgabe, die Ihr bearbeiten sollt ist:

Theorem: Die Wurzel von 2 ist nicht rational.

Beweisskizze: Nehmen wir an, dass $\sqrt{2}$ rational ist. Dann gibt es zwei ganze Zahlen n, m , die teilerfremd sind, so dass $\sqrt{2} = \frac{n}{m}$. Dann gilt auch $2 * m^2 = n^2$ (1), womit n^2 eine gerade Zahl ist. Nach dem Satz über die eindeutige Primzahlfaktorisation ganzer Zahlen muss dann auch n gerade sein. Damit gilt: $n = 2 * k$ (2) für eine ganze Zahl k . (2) eingesetzt in (1) ergibt $2 * m^2 = 4 * k^2$ was sich zu $m^2 = 2 * k^2$ kürzen lässt. Damit sind dann auch m^2 und m gerade Zahlen. Somit haben wir, dass sowohl n als auch m gerade sind. Dies ist ein Widerspruch dazu, dass n, m teilerfremd sind.

Was genau ist zu tun?

Ihr sollt die schrittweise Erstellung dieses Beweises durchspielen und dabei einige Zustände eures GUI vorbereiten. Wir unterscheiden dabei explizit zwischen GUI Zustand und Interaktion. Der schrittweise Beweisaufbau besteht aus einer wechselnden Folge von GUI Zustand und Interaktion. Jeder GUI Zustand soll auf einem extra DIN A2 Papier vorbereitet werden. Die anschließende Interaktion soll dann an diesem DIN A2 Papier durchgeführt werden. Dazu muss dann auch entsprechendes Material vorbereitet werden, das ihr bei der Vorführung auf das DIN

A2 Papier pinnen könnt (alles wird an Korkwand vorgeführt) um eure Interaktionen darlegen zu können.

Allgemeiner Hinweis: Beweismanipulation

Es gibt allgemein zwei Möglichkeiten einen Beweis zu manipulieren:

1. Das System stellt Operatoren zur Verfügung, die angewandt werden können (wir vermeiden jetzt mal den Begriff Methode oder Taktik). Dieser Operator bekommt einen bestimmten Input und liefert dann dazu einen bestimmten Output. Zum Beispiel der Operator `DEFINITIONEXPANSION` wird auf das Vorkommen eines definierten Konzeptes in einer Aussage angewandt um das definierte Konzept zu expandieren. D.h. er bekommt als Input das konkrete Vorkommen des Konzeptes in einem Ausdruck sowie die Definition des Konzeptes. Er liefert als Output den entsprechenden Ausdruck, in dem dieses Vorkommen des Konzeptes expandiert ist.
2. Unabhängig von Operatoren können auch jederzeit Beweisteile skizziert werden, indem Aussagen frei eingefügt und verlinkt werden. Etwa wie folgt:
 - (vorwärts) Ich füge Aussage A in meinem Beweis ein und A soll folgen aus bereits gegebenen Aussagen A_1 und A_2 .
 - (rückwärts) Ich muss Aussage A zeigen und A soll folgen aus neuen Aussagen A_1 und A_2 , die ich in meinen Beweis einfüge.

Ihr solltet davon ausgehen, dass immer eine Vielzahl von Operatoren zur Verfügung stehen (entsprechend einer größeren Menge von Methoden oder Taktiken). Es kommt aber vor, dass eventuell keine Operatoren da sind, die jetzt genau die Schritte ausführen, die ihr gerne hättet. Dann könnt ihr eure Schritte (d.h. euren Beweis) nur über frei eingegebene Aussagen ausdrücken.

Allgemeiner Hinweis: Funktionalitäten des Beweissystems

Das darunterliegende Beweissystem stellt eine Menge von Basis-Funktionalitäten zur Verfügung, z.B.:

- Operator anwenden (sofern anwendbar)
- Test von Anwendbarkeit von Operator
- Generierung von Vorschlägen (= Liste von anwendbare Operatoren)
- Instantiierung von Variablen
- Einfügen und Verlinken von Aussagen (ohne Operatoren) (beinhaltet z.B. auch den Check ob Zykel entstehen)
- Angabe des Beweiszustandes (Was für Ziele? Was für Annahmen? Was für Annahmen relevant für welche Ziele? ...)

- ...

Hinweis: Diese Liste der Funktionalitäten ist nicht exklusiv. Es steht euch frei euch noch beliebige weitere Funktionalitäten zu überlegen und einzusetzen (z.B. Anfrage an Auto-Mode: Was würdest Du als nächstes machen? etc.). Ihr dürft auch hier kreativ sein!

4. Schrittweise Erstellung des Beweises

Folgenden Beweisaufbau sollt ihr nun konkret durchdenken und entsprechendes Material für die Präsentation vorbereiten.

GUI Zustand 1: Das Problem “ $\sqrt{2}$ ist nicht rational” wurde geladen. Wie sieht euer GUI aus?

Interaktion 1: Ihr entschließt euch einen indirekten Beweis zu führen. Dafür steht der Operator INDIREKT zur Verfügung. Wie wollt ihr diesen Operator im GUI anwenden? Die Anwendung dieses Operators gelingt. Was soll der GUI für Feedback geben?

GUI Zustand 2: Für den indirekten Beweis muss angenommen werden, dass $\sqrt{2}$ rational ist um dann anschließend einen Widerspruch herzuleiten. Wie sieht euer GUI aus?

Interaktion 2: x ist-rational ist ein definiertes Konzept. Die Definition sagt, dass es für x eine Darstellung als Bruch zweier teilerfremden ganzen Zahlen gibt. Ihr wollt diese Definition aus einer angeschlossenen Datenbank erfragen. Wie wollt ihr das im GUI machen? Was soll der GUI an Output oder weiterem Feedback liefern?

Ihr versucht (fehlerhafterweise) diese Definition mittels des Operators DEFINITIONEXPANSION auf $\sqrt{2}$ anzuwenden. Wie wollt ihr das im GUI machen? Diese Anwendung des Operators schlägt fehl. Was soll der GUI an Output oder weiterem Feedback liefern?

Anschließend wendet ihr diese Definition auf $\sqrt{2}$ ist-rational an (das in irgendeiner Form in eurem Beweis vorkommen muss). Wie wollt ihr das im GUI machen? Diese Anwendung des Operators gelingt. Was soll der GUI für Feedback geben.

GUI Zustand 3: Das Resultat dieser Definition Expansion muss irgendwie $\sqrt{2} = \frac{n}{m}$ enthalten. Wie sieht euer GUI aus?

Interaktion 3: Ihr wollt $2 * m^2 = n^2$ als frei spezifizierte Aussage einführen und spezifizieren, dass es aus $\sqrt{2} = \frac{n}{m}$ folgen soll. Wie wollt ihr das im GUI machen? Was soll der GUI an Feedback liefern?

GUI Zustand 4: Anschließend leitet ihr in einer Folge von frei spezifizierten (aber verlinkten) Aussagen ab, dass sowohl n als auch m gerade sind. Wie sieht euer GUI aus?

Interaktion 4: Ihr leitet den Widerspruch her aus den (inzwischen in irgendeiner Form enthaltenen) Aussagen, dass n und m gerade sind sowie, dass n und m teilerfremd sein sollen. Dies schließt euren Beweis. Wie wollt ihr das im GUI machen? Was soll der GUI an Feedback liefern?

Hinweis: Erst ein GUI Gesamtkonzept überlegen, in dem sich diese Sachen realisieren lassen. Dann erst anfangen zu basteln!

5. Präsentation

Bei der Präsentation soll neben dem Durchspielen des Beweises insbesondere Folgendes erklärt werden:

- Was sind die grundlegenden Elemente des GUI's?
- Was bedeuten diese Elemente?
- Wie und wo wird der momentane Beweiszustand dargestellt?
- Wie und wann werden Funktionalitäten des darunterliegenden Beweissystems benutzt und wie werden diese Funktionalitäten über die GUI Elemente angesprochen?

Noch Fragen?

Um es nochmal zu sagen: Nicht an Ω MEGA Erfahrung anlehnen (oder an irgend ein anderes Tool, das ihr kennt). Lasst eurer Kreativität freien Lauf!

Viel Spaß!

B Proposal Fotos

We have attached some fotos, taken during the presentation of each aproach. They essentially show the structure of a GUI window, a proof presentation and the application of operators.

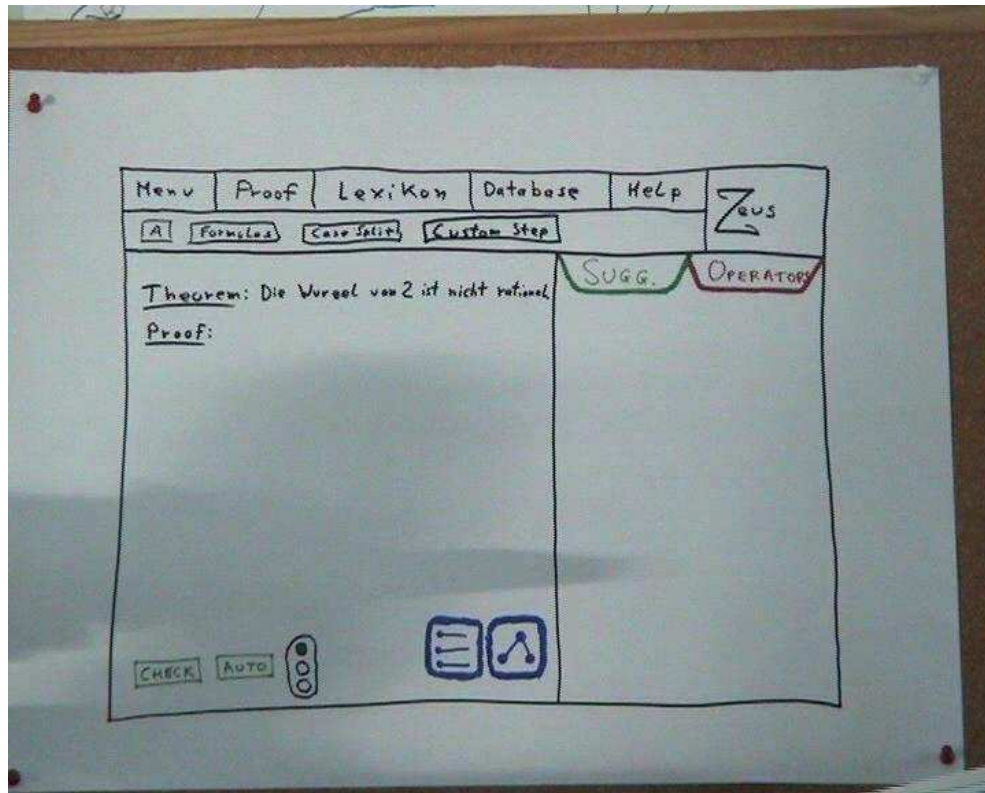


Figure 10: GUI window (Group A)

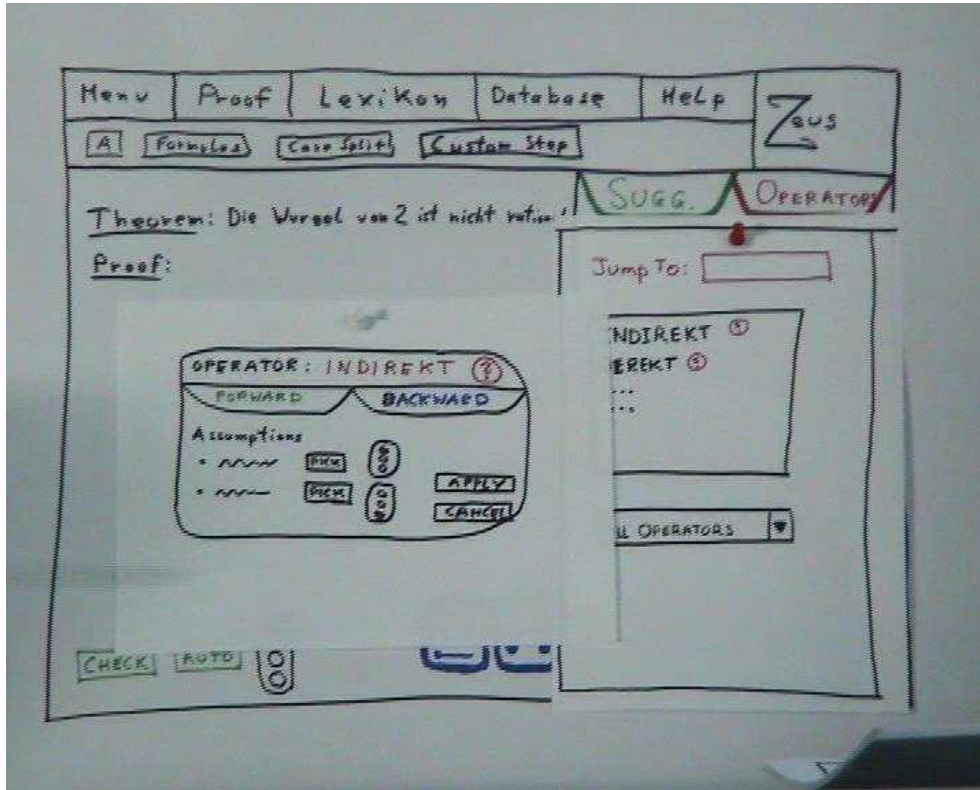


Figure 11: Operator application (Group A)

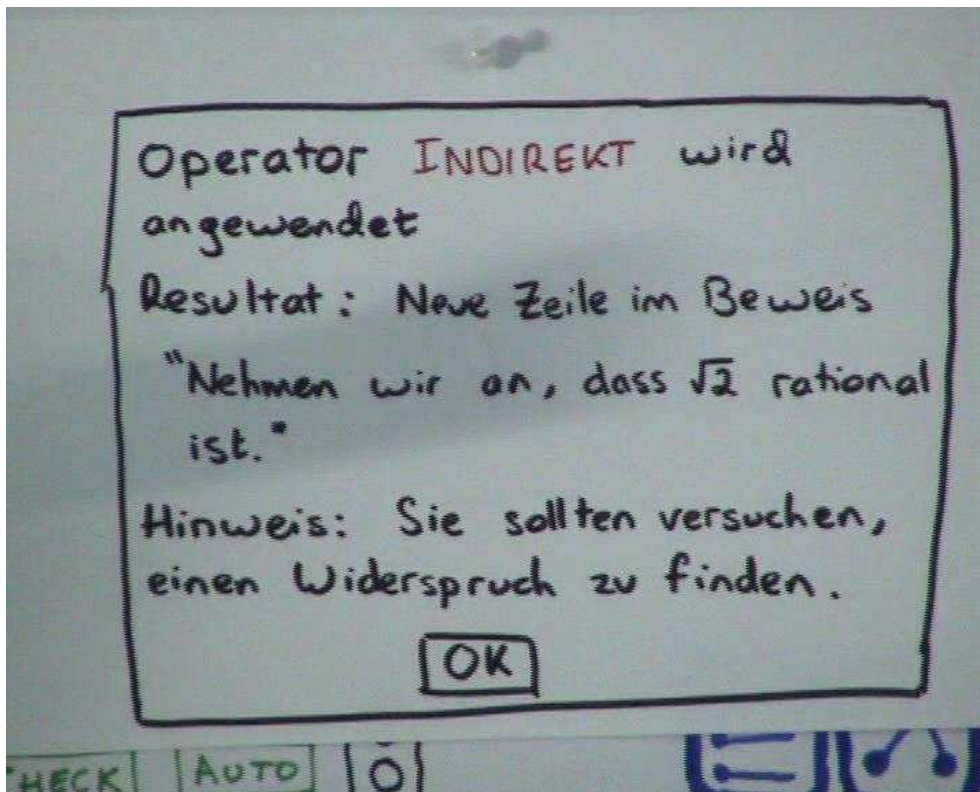


Figure 12: Operator application feedback (Group A)

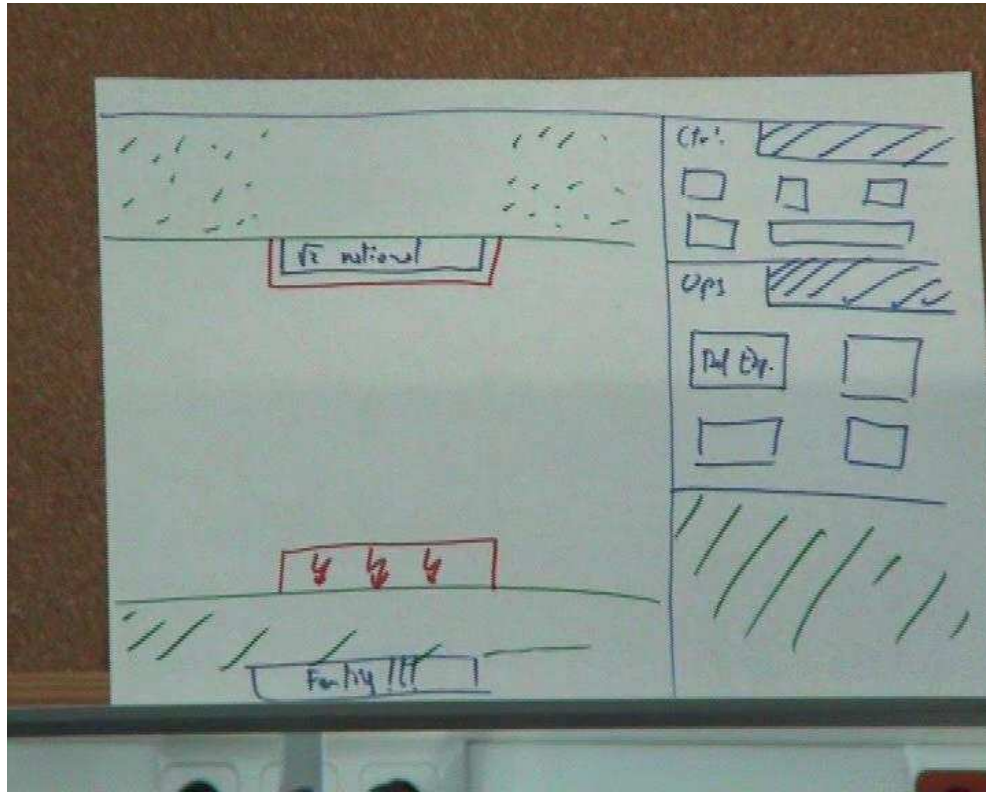


Figure 13: GUI window (Group B)

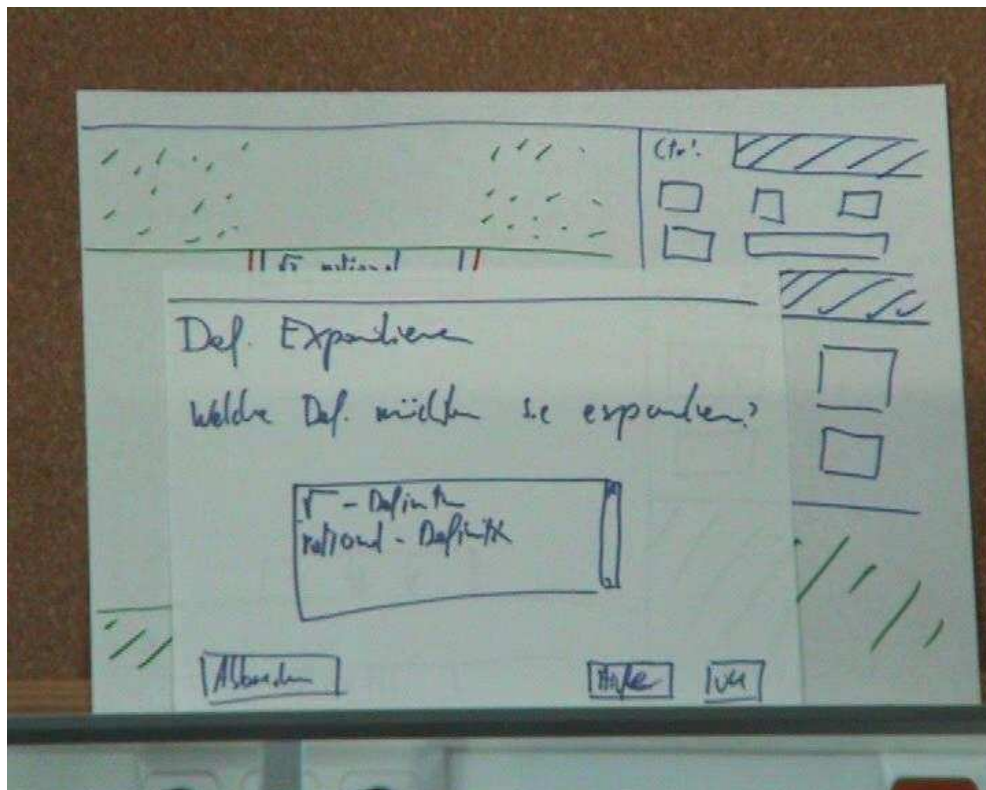


Figure 14: Operator application (Group B)

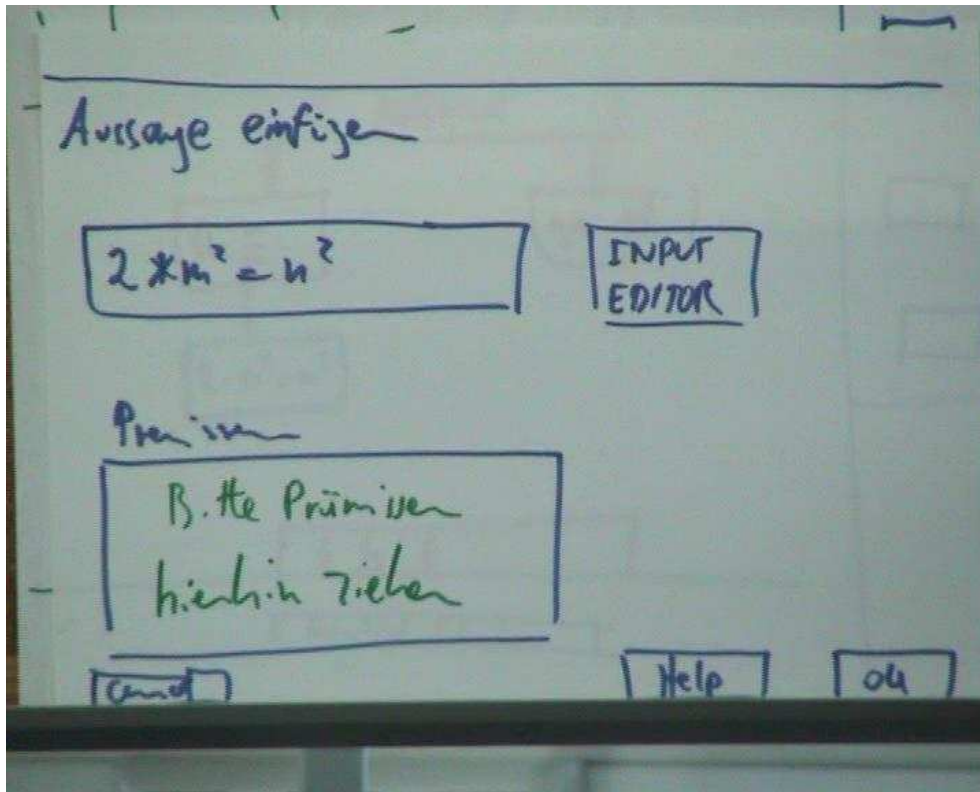


Figure 15: Island insertion (Group B)

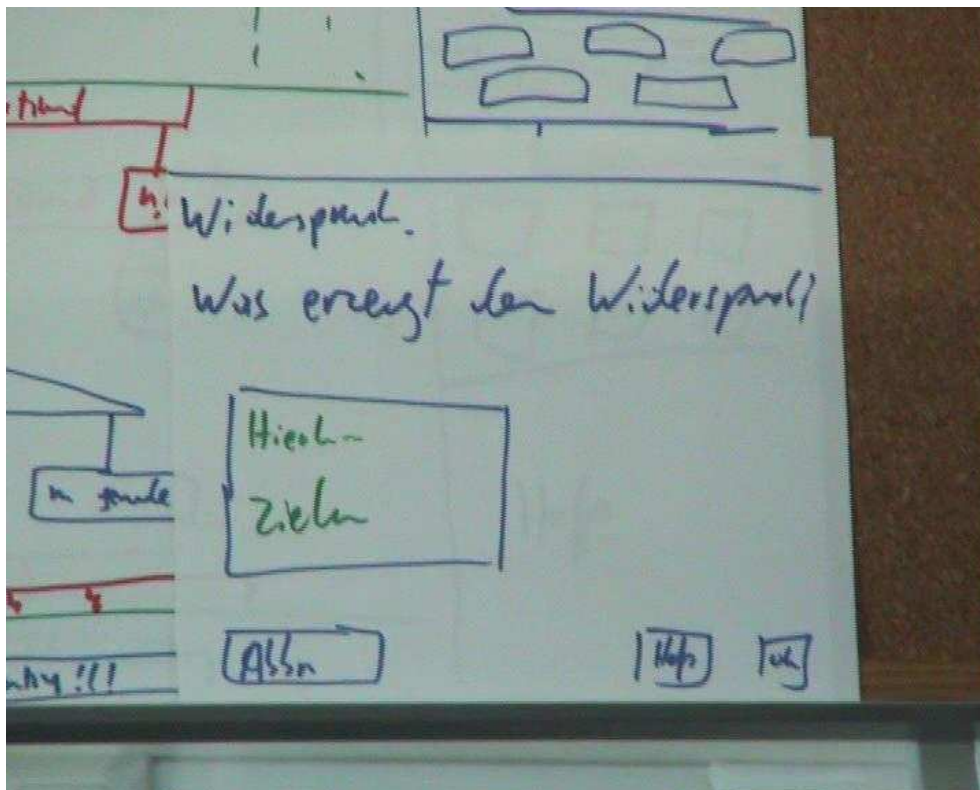


Figure 16: Contradiction (Group B)

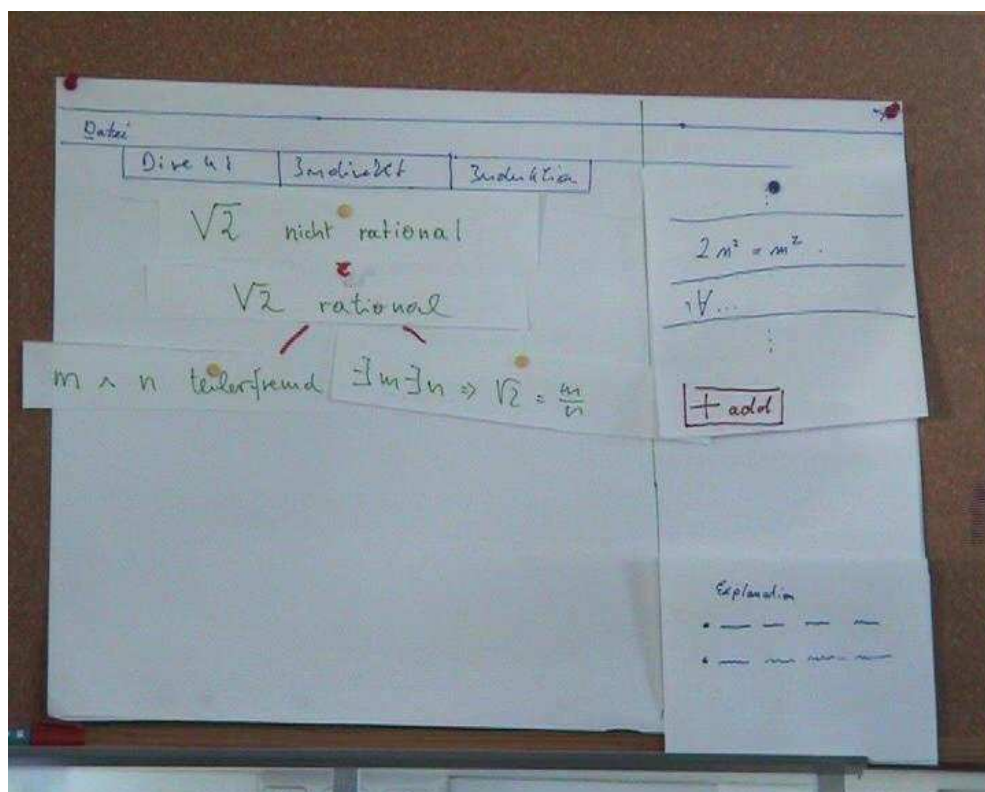


Figure 17: GUI window (Group C)

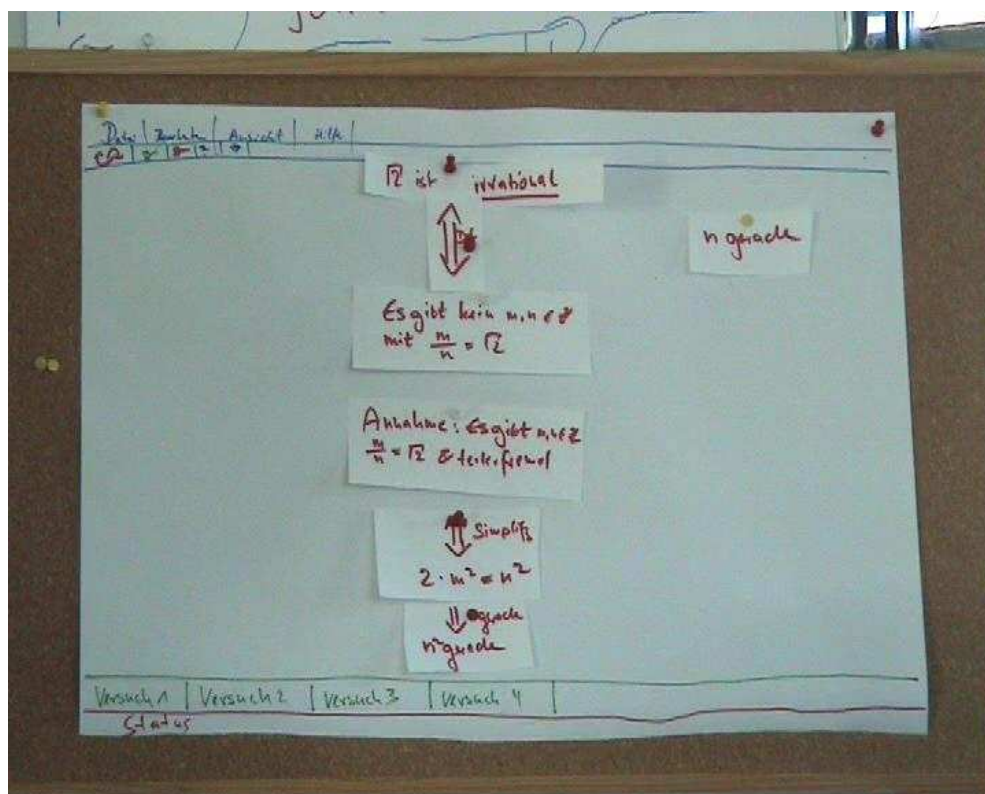


Figure 18: GUI window (Group D)

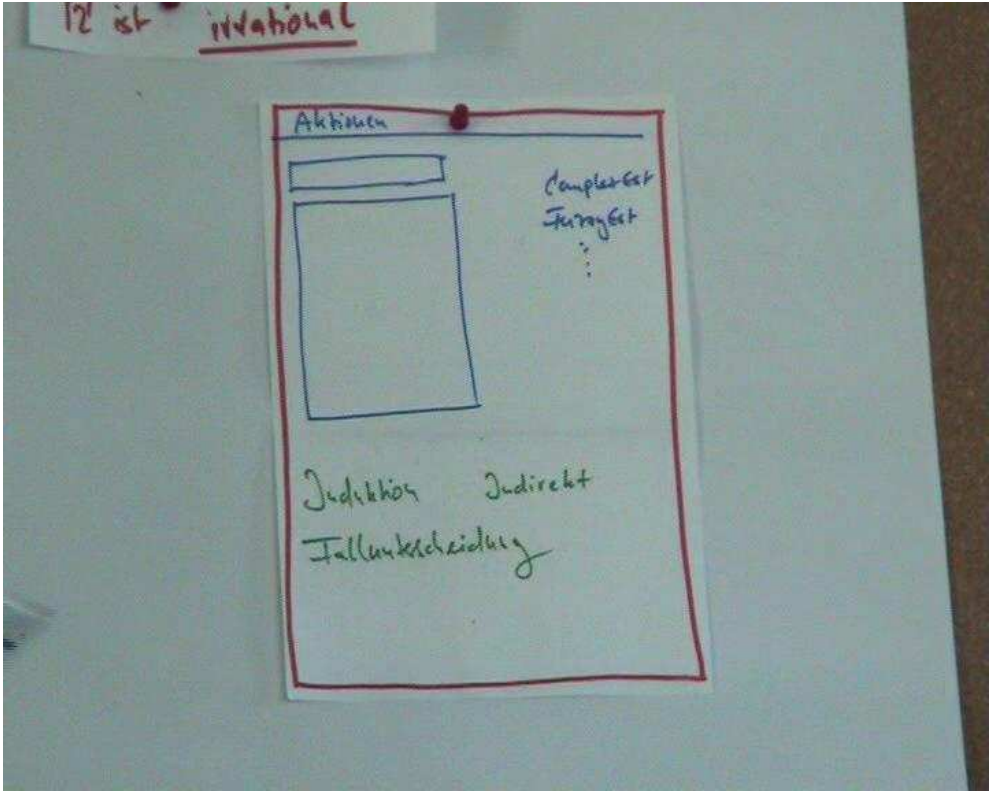


Figure 19: Operator (Group D)